Expected Value

Example 1: I have a spinner. It has 5 colors:

Red has 100 degrees and you win \$2

Green has 60 degrees and you lost \$2

Yellow has 65 degrees and you win \$1

Blue has 90 degrees and you lose \$1

Orange has 45 degrees and you win \$4

- What colors do you expect to spin the most? Least?

Red, Orange

- Overall for this spinner, do you expect to win money or lose money? Why?

255° +

- Create a probability model for this spinner

X	\$2	-\$1	-\$2	\$4	\$1
P(x)	100/360	90/360	60/360	45/360	65/360

- Find the expected value of the spinner

\$0.65

- The expected value is...

- So if I wanted to find out how much I EXPECT to win on 65 spins, I would....

$\$0.65 \times 65 = \$42.25$

EXPECTED VALUE:

What is it?

- Basically … long-run average
- Also called… mean
- Symbol: $E(X)$ ← game, trial

Formula (how to find it):

- **X is a variable with the following probability model:**

X	X1	X2	X3 …	Xn
P(X)	P(X1)	P(X2)	P(X3) …	P(Xn)

- **Find the mean (expected value) by doing the following:**

$$\sum X \cdot P(X)$$

Example:

L_1= X	0	1	2	3	4	5
L_2= P(X)	0.05	0.12	0.18	0.2	0.4	0.05

What is the mean (expected value)?

$\boxed{E(x) = 2.93}$ $(0 * 0.05) + (1 * 0.12$

- Using the same data, use your calculator to find the Expected Value (mean):
  - **Data goes in…** L_1 = x-values, L_2 = prob.
  - **Use:** 1-var stats L_1, L_2
  - **Mean:** $\bar{x}$

Example 2: I have an unfair dice. It has the following probabilities:

15% chance of getting 6 — 20% chance of getting 5

10% chance of getting 4 — 12% chance of getting 3

30% chance of getting 2 — 13% chance of getting 1

- Create a probability model for this dice

X	1	2	3	4	5	6
$P(X)$	0.13	0.30	0.12	0.10	0.20	0.15

- Find the expected value of this dice

$E(X) = 3.39$

- If I rolled the dice 58 times, what would I expect the average of those rolls to be?

$3.39 \times 58 = 196.62$

Example 3:

At many airports, a person can pay only $1.00 for a $100,000 life insurance policy covering the duration of the flight. In other words, the insurance company pays $100,000 if the insured person dies from a possible flight crash; otherwise the company gains $1.00 (before expenses). Suppose that past records indicate 0.45 deaths per million passengers.

- Create the probability model for the **GAIN** of the insurance company on each policy

X	-$99,999	$1
P(X)	$\frac{0.45}{1{,}000{,}000}$	$\frac{.55}{1{,}000{,}000}$

- How much can the company expect to gain on one policy?

- How much can the company expect to gain on 100,000 policies?

Complete the worksheet (#1 -- 6)

Worksheet answers:

1) E(X) = 7.86

2) a

X	60,000	45,000	15,000
P(x)	1/2	1/3	1/6

b $E(x) = 47{,}500$

3) a

X	\$74,999	-\$1
P(x)	$\frac{1}{10{,}000}$	$\frac{9{,}999}{10{,}000}$

b $E(x) = \$6.50$

4 a

X	\$74,999	\$999	\$99	−\$1
P(x)	$\frac{1}{10,000}$	$\frac{5}{10,000}$	$\frac{10}{10,000}$	$\frac{9984}{10,000}$

b $E(x) = \$7.10$

5 a

A

X	250,000	−10,000
P(x)	0.1	0.9

B

X	40,000	−2,000
P(x)	0.5	0.5

C

X	800,000	−20,000
P(x)	0.05	0.95

#6) Game 1

X	-1	0	1
P(x)	0.40	0.10	0.50

\$0.10

Game 2

X	-1	0	1
P(x)	0.05	0.80	0.15

\$0.10

HW ANSWERS:

4) a) $0.0429 = P(F \cap H)$

b) $P(F \cup H) =$

$P(F) + P(H) - P(F \cap H)$

$0.33 + 0.28 - 0.0429$

$= 0.5671$

9 $P(R) = 0.37$

$P(J \cap R) = 0.15$

$\cancel{P(R|J)} = \underline{\qquad}$

$$P(J|R) = \frac{0.15}{0.37} = 0.4054$$

10 $P(m) = 0.50$

$P(m \cap J) = 0.20$

$$P(J|M) = \frac{P(J \cap M)}{P(M)} = 0.40$$

8 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.78 = 0.25 + P(B) - 0.12$

13 $(40 \text{ nCr } 4) \cdot (65 \text{ nCr } 4) \cdot (50 \text{ nCr } 4)$

$1.425 \ldots\ldots E\ 16$

1.425×10^{16}